Rewarding Learning

ADVANCED
General Certificate of Education
2022

Further Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

AFM11

[AFM11]
WEDNESDAY 25 MAY, MORNING

TIME

2 hours 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all twelve** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all twelve questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Find

$$\sum_{r=1}^{n} r^2 (r+3)$$

in terms of n. [5]

(ii) Hence find the sum of the series

$$5^2 \times 8 + 6^2 \times 9 + \cdots + 40^2 \times 43$$ [3]

2 Using the principle of mathematical induction, prove that for all integers $n \geqslant 1$

$$5^{2n} - 1 \quad \text{is divisible by } 8$$ [7]

3 **(a)** Find

$$\int e^x \cos x \, dx$$ [8]

(b) Evaluate

$$\int_5^{\infty} \frac{1}{(x-3)^2} \, dx$$ [4]

4 The graph of $r = 4 \sin 2\theta$ is shown in **Fig. 1** below.

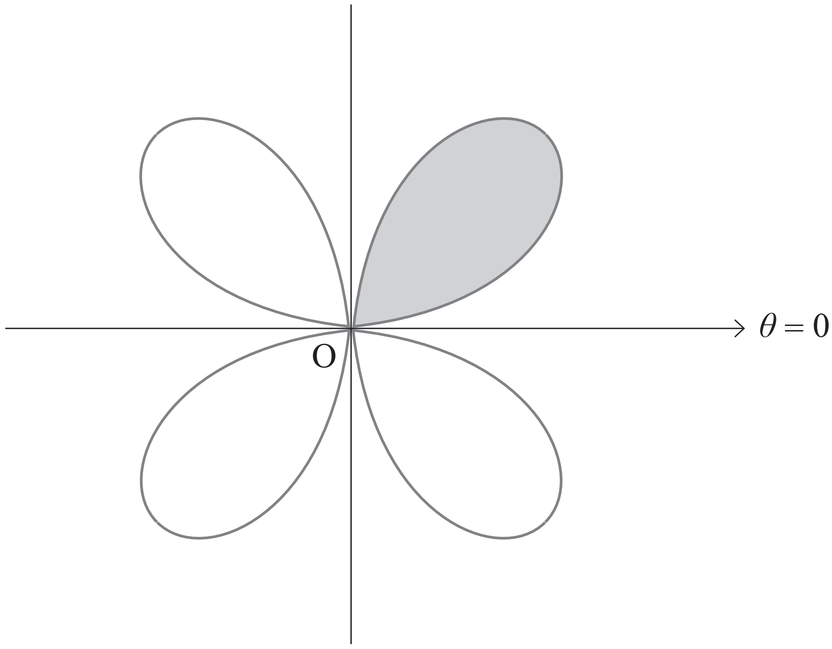

Fig. 1

Find the area of the shaded region. [8]

5 **(i)** Express

$$\frac{3r - 1}{r^3 - r}$$

in partial fractions. [6]

(ii) Hence, or otherwise, prove that

$$\sum_{r=2}^{n} \frac{3r - 1}{r^3 - r} = 2 - \frac{1}{n} - \frac{2}{n + 1} \quad \text{for } n \geqslant 2$$ [5]

6 **(i)** Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan^{-1}x + \frac{x}{1+x^2}\right] = \frac{2}{(1+x^2)^2} \qquad [5]$$

(ii) Hence find the exact value of

$$\int_3^4 \frac{1}{(x^2 - 6x + 10)^2}\,\mathrm{d}x \qquad [6]$$

7 Solve the differential equation

$$(x^2 - 1)\frac{\mathrm{d}y}{\mathrm{d}x} + 4xy = 8x \quad \text{where } |x| > 1$$

given that $y = 5$ when $x = 2$ [10]

8 **(i)** If

$$\mathrm{f}(x) = (1 + \cos x)\,\mathrm{e}^x$$

show that

$$\mathrm{f}''(x) = (1 - 2\sin x)\,\mathrm{e}^x \qquad [4]$$

(ii) Use Maclaurin's theorem to derive the series expansion for

$$\mathrm{f}(x) = (1 + \cos x)\,\mathrm{e}^x$$

up to and including the term in x^4 [7]

(iii) Hence, or otherwise, find the series expansion for

$$(1 + \cos x)\,\mathrm{e}^{-x}$$

up to and including the term in x^4 [4]

9 **(i)** Given that

$$I_n = \int \sec^n x \, \mathrm{d}x$$

show that for $n \geqslant 2$

$$(n-1)I_n = \sec^{n-2}x \tan x + (n-2)I_{n-2} \qquad [8]$$

(ii) A vase can be modelled by rotating the curve

$$y = \sec^2 x$$

through 2π radians about the x-axis, between the ordinates $x = 0$ and $x = \frac{\pi}{3}$

Using part **(i)** find the volume of the vase. [6]

10 A particle moves so that at time t seconds its displacement x metres, from the origin O, is given by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - \frac{\mathrm{d}x}{\mathrm{d}t} - 6x = 52\cos 2t$$

(i) Find the general solution of the differential equation. [13]

It is known that the particle's displacement remains finite as $t \rightarrow \infty$ and
$x = 1$ when $t = 0$

(ii) Find the displacement x of the particle as a function of t. [3]

11 **(a)** Find the equation of the tangent to the curve

$$y = \sinh^{-1}(2x - 1) + 4x$$

at the point where $x = \frac{1}{2}$ [7]

(b) **(i)** Show that

$$\sinh^{-1}x \equiv \ln\left(x + \sqrt{x^2 + 1}\right)$$ [5]

(ii) Hence find the exact solution of the equation

$$\sinh^{-1}x + \sinh^{-1}\left(\frac{4}{3}\right) = 2\sinh^{-1}\left(\frac{3}{4}\right)$$ [7]

12 **(a)** Find, in the form $re^{i\theta}$, the values of the 6 roots of the equation

$$z^6 + 64 = 0$$

and plot them on a carefully labelled Argand diagram. [8]

(b) Let $z = \cos\theta + i\sin\theta$

(i) Show that $z + \frac{1}{z} = 2\cos\theta$ [3]

(ii) Hence find an expression for $\cos^4\theta$ in the form

$$p\cos 4\theta + q\cos 2\theta + r$$ [6]

(iii) Hence find

$$\int \cos^4\theta \, d\theta$$ [2]

THIS IS THE END OF THE QUESTION PAPER